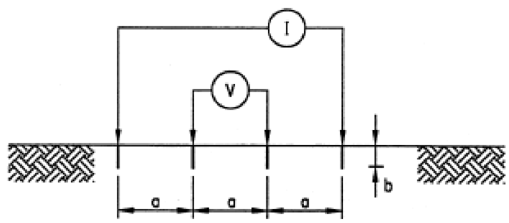
CALCULATION OF THE OPTIMIZED PARAMETERS OF A MULTILAYER SOIL MODEL STARTING FROM ON SITE MEASUREMENTS



Wenner arrangement for soil resistivity measurements

The **measured apparent soil resistivity** can be evaluated simply with the formula (see figure):

$$\rho(a) = 2\pi a \frac{V}{I}$$

where:

- a (m) = inter electrode spacing (see figure)
- V (V) = measured voltage
- I (A) = measured current

Changing the inter electrode spacing, we can investigate the soil at different depth. Results can be used to evaluate the optimized parameters of a multilayer soil model.

The **calculated apparent soil resistivity of a multilayer soil** can be evaluated with the formula:

$$\rho_c(a, \rho_1, \rho_2, \cdots \rho_n, h_1, h_2, \cdots h_{n-1}) = \rho_1 + 4\rho_1 a \int_0^\infty B(\lambda)[J_0(\lambda a) - J_0(2\lambda a)]d\lambda$$

where:

- a (m) = inter electrode spacing (see figure)
- ρ_i (Ωm) = soil resistivity of the layer i
- h_i (m) = thickness of the layer i
- ρ_1 (Ωm) = soil resistivity of the first (upper) layer
- $J_0(\lambda a)$ = Bessel function first kind and zero order
- $B(\lambda)$ = kernel function

The kermel function depends on the layers number.

In the following, we consider for instance a 3 layer soil model, so:

$$B_3(\lambda) = \frac{K_{31} e^{-2\lambda h_1}}{1 - K_{31} e^{-2\lambda h_1}}$$

$$K_{31} = \frac{\upsilon_{12} + \upsilon_{23} e^{-2\lambda h_2}}{1 + \upsilon_{12} \upsilon_{23} e^{-2\lambda h_2}}$$

$$\upsilon_{12} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$\upsilon_{23} = \frac{\rho_3 - \rho_2}{\rho_3 + \rho_2}$$

The thickness of the third (bottom) layer is infinite, so **in case of a 3 layers the soil parameters number is 5**.

The **apparent soil resistivity can be calculated only with numerical integration**. We use the Simpson formula.

The final task is the calculation of the optimized 5 parameters starting on "m > 5" apparent soil resistivity measured values with different inter electrode spacing.

This is a typical nonlinear least square problem.

The optimized 5 parameter are the parameters which lead to the minimum of the following squared error function:

$$\psi(\rho_1, \rho_2, \rho_3, h_1, h_2) = \sum_{i=1}^{m} \left[\frac{\rho(a_i) - \rho_c(a_i, \rho_1, \rho_2, \rho_3, h_1, h_2)}{\rho(a_i)} \right]^2$$

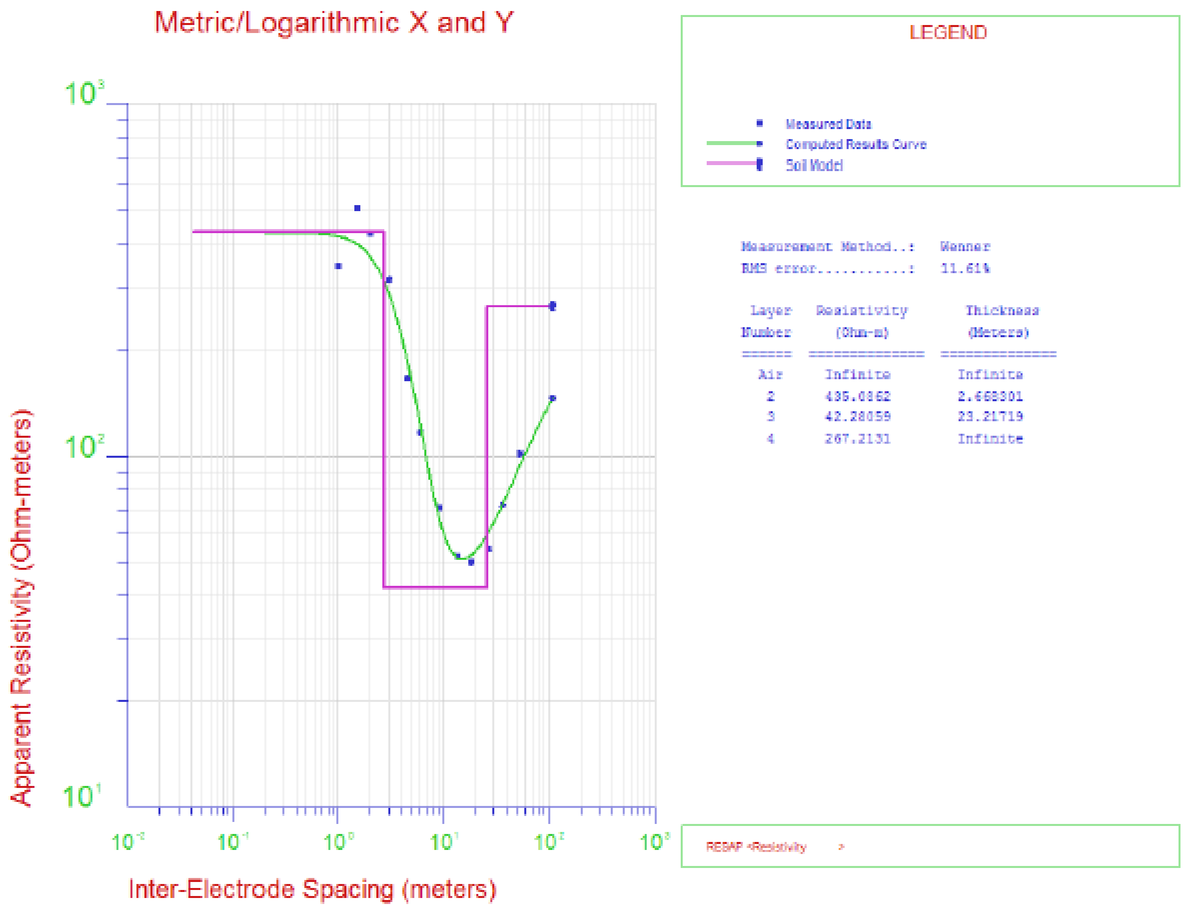
where:

- $\rho(a_i)$ (Ωm) = measured apparent soil resistivity (set of "m" measurements)
- $\rho_c(a_i)$ (Ωm) = calculated apparent soil resistivity
- a_i (m) = inter electrode spacing

Using the MKL trust region method, the "m" function values are:

$$fvec[i] = \psi_i(\rho_1, \rho_2, \rho_3, h_1, h_2,) = \rho(a_i) - \rho_c(a_i, \rho_1, \rho_2, \rho_3, h_1, h_2) = 0 \quad \text{i = 1 … m}$$

Our competitors for the same problem use the Levemberg Marquardt method. The following figures shows a typical situation: blue points represents “m” measurements, the green curve represents the calculated apparent resistivity with the optimized 5 parameters, the violet lines represents the corresponding soil model (then 5 parameters).



Typical situation